Analysis & Design of Algorithm **MCA – 4**

Full Marks : 70
Time : 3 hours

Answer any <u>six</u> questions including **Q.No.1** which is compulsory.
The figures in the right-hand margin indicate marks.

1. Answer all questions. [2 x 10]

(a) If $T(n) = 2T(n/2) + n^3$, then find $O(T(n))$.

(b) Is the sequence < 23, 17, 14, 6, 13, 10, 1, 5, 7, 12 > a max-heap ? Justify.

(c) Let X be a random variable that is equal to the number of heads in two flips of a fair coin. What is $E[X^2]$? What is $E^2[X]$?

(d) Prove that for any pair of vertices 'u' and 'v' and any capacity and flow functions 'c' and 'f', we have $c_f(u,v) + c_f(v,u) = c(u,v) + c(v,u)$.

(e) What are approximation algorithms ? What is their significance ?

(f) If running time of one algorithm is $O(n \log n)$ and another is $O(n^3)$, then which one is more efficient and why ?

(g) Define the convex hull of a set of points. Give an example.

(h) What are the three properties that a flow in a flow network must satisfy ?

(i) Explain the fractional knapsack problem with a suitable example.

(j) What do you mean by a NP-complete problem ? Give an example.

2. (a) Illustrate the operation of BUILD-MAX-HEAP on the array
A = < 5, 3, 17, 10, 84, 19, 6, 22, 9 >. [5]

(b) Discuss the running time of quick sort algorithm with respect to balanced versus unbalanced partitioning of elements. [5]

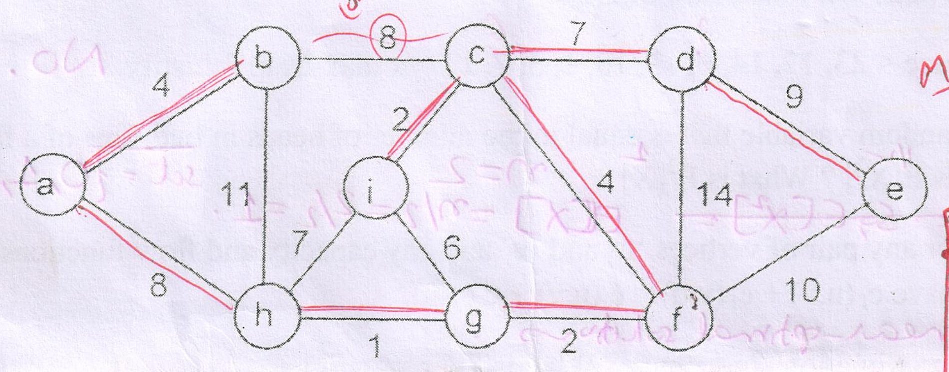
3. (a) What is the use of indicator random variables in randomized algorithms ? Explain with a suitable example. [5]

(b) Differentiate between aggregate analysis, accounting and potential methods of amortized analysis with suitable examples. [5]

4. (a) What are the elements of dynamic programming ? Determine a Longest Common Subsequence of < 1, 0, 0, 1, 0, 1, 0, 1 > and < 0, 1, 0, 1, 1, 0, 1, 1, 0 > using dynamic programming. [5]

(b) Explain with a suitable example why the greedy strategy doesn't work for the 0/1 knapsack problem. [5]

5. (a) Find a minimum spanning tree of the following graph using Kruskal's algorithm. Show all the steps and find the total cost of the minimum spanning tree. [5]



(b) Explain how to find the connected components of undirected graphs with a suitable example. [5]

6. (a) What is backtracking technique of analyzing algorithms ? Discuss the 4-queens problem and its solution using backtracking technique. [5]

(b) How is the Dijkstra's algorithm for the single source shortest path problem different from the Bellman-Ford algorithm ? Explain with suitable examples. [5]

7. (a) Explain the Rabin-Karp string matching algorithm with a suitable example. [5]

(b) Explain the Graham's scan algorithm to find the convex hull of a given set of points with a suitable example. [5]

8. (a) Explain the divide-and-conquer technique of analyzing algorithms with a suitable example. [5]

(b) Define the formula-satisfiability problem and prove that it is NP-complete with a suitable example. [5]